

Stig Rehnmark
Anaren Microwave, Inc.
185 Ainsley Drive
Syracuse, N.Y. 13205

Abstract

A very powerful method to measure the parameters of a one-section Coupled Transmission Line (CTL) coupler is presented. Three transmission measurements at the frequency of maximum coupling is sufficient to give the even and odd mode impedances (Z_e , Z_o) and the even and odd mode lengths (θ_e , θ_o). Explicit equations are presented for the necessary calculations.

A new method for wideband improvement of CTL-coupler directivity when $\theta_e \neq \theta_o$ has been developed. The theory is presented with all necessary equations and it covers both $\theta_e > \theta_o$ (microstrip) and $\theta_e < \theta_o$. The new method makes use of short compensating sections in the coupler to achieve a directivity pole at an arbitrary frequency. The wanted bandwidth determines the optimum location of the pole. Several examples with design data are given.

Introduction

It is well known that a Coupled Transmission Line (CTL) coupler with unequal even and odd mode lengths suffers from low directivity. A length difference between the even and odd mode can be caused by:

- (a) Different dielectric layers (ex., microstrip)
- (b) Inhomogeneous dielectric (ex., glassfibers in teflon)
- (c) Rough Cu surface (ex., electro deposited Cu on teflon)
- (d) Corner effects where the two coupled lines are launched
- (e) Different "paths" of the even and odd mode.^{1,2}
- (f) Meander-folding of coupled lines in homogeneous dielectric.^{2,3}

Methods to improve the directivity of microstrip couplers (i.e., the even mode is longer than the odd mode) has been suggested by Podell⁴, Buntschuh⁵ and others³. Little has been presented for the case when the even mode is shorter than the odd mode. The latter case is, however, very important since that is what we measure for "homogeneous" dielectric (commercially available glassfiber reinforced teflon) stripline couplers. The work presented here is a part of a research program at Anaren Microwave, Inc., with the following goals:

- (1) Develop a method to measure the even and odd mode impedances (Z_e , Z_o) and the even and odd mode lengths (θ_e , θ_o) of a single section CTL-coupler.
- (2) Measure the length ratio of the two modes (θ_e/θ_o) as a function of frequency and coupling.
- (3) Determine the cause of the length difference.
- (4) Develop a method to achieve high directivity couplers.

The main purpose of this paper is to present the results from Parts 1 and 4 above.

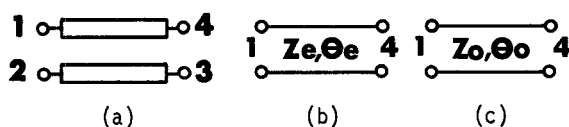


Figure 1 - The coupled transmission line coupler

- (a) Total Circuit
- (b) Even Mode Circuit
- (c) Odd Mode Circuit

A Measurement Method of CTL-Coupler Parameters

A CTL-coupler is characterized by the even and odd mode impedances (Z_e , Z_o) and the even and odd mode lengths (θ_e , θ_o) as is shown in Figure 1.

The presented measurement method has been used to measure the parameters of numerous experimental couplers and then optimizing the performance. The method requires three measurements at a well specified frequency. The procedure is as follows:

- (1) Determine the frequency where the coupling is maximum
- (2) Measure at that frequency, with the coupled port as reference:
 - (a) DIR = directivity = $20 \log (|S_{31}|/|S_{21}|)$
 - (b) ADIR = angle of directivity = $\angle S_{31} - \angle S_{21}$
 - (c) CREL = relative coupling = $20 \log (|S_{41}|/|S_{21}|)$
- (3) DIR, ADIR, CREL are then input data in a computer program that gives Z_e , Z_o , θ_e and θ_o .

S_{21} , S_{31} and S_{41} are the scattering parameters of the coupler.

Explicit formulas for the calculation of Z_e , Z_o , θ_e and θ_o have been found for the two special cases $Z_e \cdot Z_o = 1$, (ADIR = 0 or 180 degrees) resp. $\theta_e = \theta_o$ (ADIR = ± 90 degrees). An approximate solution for the general case is found by using the projection of the directivity vector on the axis where ADIR = 0 resp. on the axis where ADIR = 90 degrees. The equations are:

$$\Delta\phi = \cos(\text{ADIR}) \cdot 10^{\text{DIR}/20} \quad (1)$$

$$\Delta Z = \sin(\text{ADIR}) \cdot 10^{\text{DIR}/20} \quad (2)$$

$$\text{CR} = 10^{\text{CREL}/20} \quad (3)$$

$$\text{CA} = (\text{CR} + \Delta Z) / (\text{CR} - \Delta Z) \quad (4)$$

$$\text{CB} = 2 \cdot (\text{CA} + 1) / \text{CR} \quad (5)$$

$$\text{H} = (1 + \text{CB} \cdot \text{CB} / 4 - \text{CA} \cdot \text{CA}) / (\text{CA} \cdot \text{CB}) \quad (6)$$

$$Z_o = \sqrt{\text{H} \cdot \text{H}^*} - \text{H} \quad (7)$$

$$Z_e = \text{CB} / 2 + \text{CA} \cdot Z_o \quad (8)$$

$$\Delta\theta = -\tan^{-1} [(Z_e + 1/Z_e) \cdot \Delta\phi / (2 \cdot \text{CR})] \quad (9)$$

$$\theta_e = 90 + \Delta\theta \quad (10)$$

$$\theta_o = 90 - \Delta\theta \quad (11)$$

The approximate method gives small errors when $\Delta\theta$ and/or $(Z_e \cdot Z_o - 1)$ are small. For other cases, a computer optimizing program has been developed to minimize the error of the calculated parameters.

Some Results From Stripline CTL-Coupler Measurements

Figure 2 shows the measured performance of a 3 dB and a 25 dB coupler compared to the theoretical performance with the measured Z_e , Z_o , θ_e and θ_o as input data. Table 1 gives the parameters used. As is seen in the figure, the agreement is very good between measured performance and calculated performance using the measured Z_e , Z_o , θ_e and θ_o .

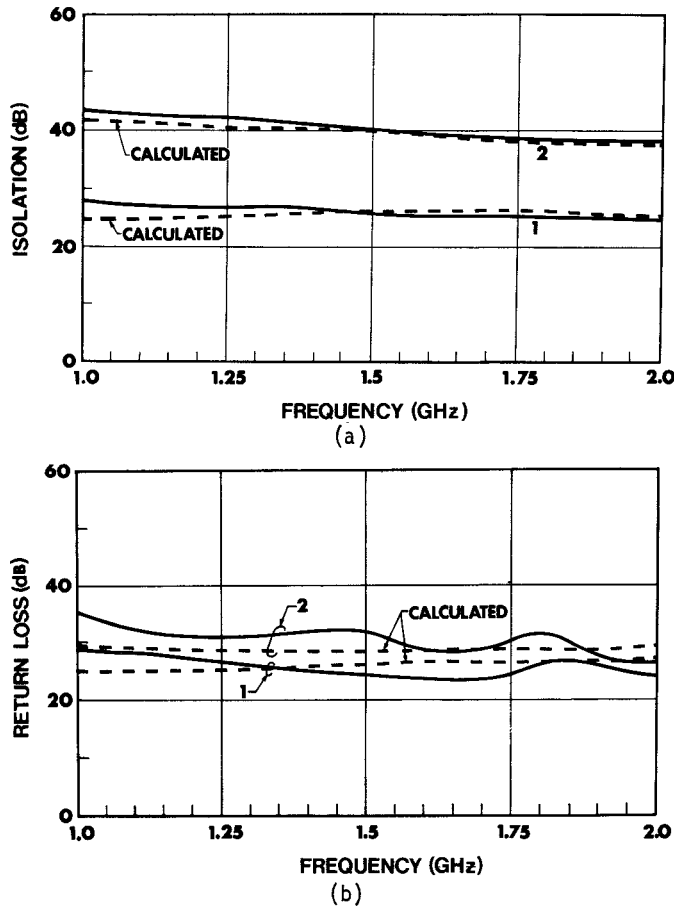


Figure 2 - Measured and calculated curves of CTL-couplers.
The parameters are found in Table 1
a. Isolation b. Return Loss

All the parameters in Table 1 represent measurements of stripline couplers. The dielectric boards are glassfiber reinforced teflon with a ground plane spacing of 65 mils, a center board thickness of 5 mils and a dielectric constant of 2.6. The coupled lines are connected to 50 ohm lines by standard 45° mitered right angle bends.

It is shown in the table that the even and odd mode length difference is large for tight coupling. By using two coupled resonant loops, with the same cross section as the 3 dB coupler in Table 1, it is possible to measure the effective dielectric constant of the even and the odd mode for that coupler. Our measurements show that the rough Cu surface (electro deposited Cu) causes an increase in the measured odd mode dielectric constant of $\approx 15\%$. No change with frequency was observed in the measured range 1-18 GHz. The even mode dielectric constant is merely unaffected by Cu roughness. (Another unwanted effect caused by the Cu roughness is an increase of the odd mode loss to several times the loss of the even mode).

Curve	DIR (dB)	ADIR (Degr)	CREL (dB)	θ_e (Degr)	θ_o (Degr)	θ_e/θ_o (%)	CPL* (dB)	IMP** (ohm)
1	-23.2	-38.2	-0.68	85.0	94.9	89.6	-2.66	0.939
-	-20.0	-38.0	2.5	85.6	94.1	91.0	-4.40	0.926
-	-21.0	-18.0	6.0	87.1	92.5	94.1	-6.96	0.970
-	-19.0	-4.5	13.8	88.6	91.2	97.1	-13.97	0.991
2	-14.8	-12.0	24.5	89.0	90.2	98.7	-24.48	0.962

$$*CPL = 20 \log[(Z_e - Z_o)/(Z_e + Z_o)]$$

$$**IMP = \sqrt{Z_e \cdot Z_o}$$

Table 1. Parameters of measured stripline couplers

For a 20 dB coupler, the Cu roughness is not as important as for a 3 dB coupler. Due to the separation of the coupled lines, a major part of the odd mode field is parallel to the ground planes and to the glassfibers. The high dielectric constant of glass makes the dielectric constant for parallel field about 15% higher than for perpendicular field. This results in a lengthening of the odd mode compared to the even mode.

A theoretical analysis of single section CTL-couplers has been made to calculate the minimum directivity over a 2.3:1 bandwidth (octave bandwidth with some margin). The results of this analysis is shown in Figure 3. The conventional coupler has the impedance level $= \sqrt{Z_e \cdot Z_o} = 1$ which will give a lower directivity at the high end of the band (solid curves). There exists an impedance, however, that will optimize the directivity over the band (dashed curves). The directivity improvement is about 2 dB when the optimum impedance level is used. See also Table 2 and Figures 6 and 7 for additional data.

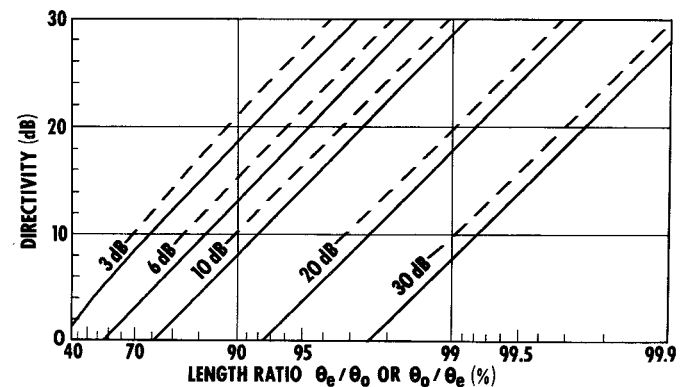


Figure 3 - Calculated minimum directivity over 2.3:1 bandwidth for single section CTL-couplers with unequal even and odd mode lengths.

—— Conventional design $Z_e \cdot Z_o = 1$
----- Optimum directivity design

Improving the Directivity of CTL-Couplers

The circuit used to improve the directivity of a coupler when $\theta_e \neq \theta_o$ is shown in Figure 4. The circuit consists of a non-commensurate, symmetrical three-section coupler with a total length $L \approx \lambda/4$.

$$\text{We now suppose that: } \frac{\theta_{e1}}{\theta_{o1}} = \frac{\theta_{e2}}{\theta_{o2}} = \frac{\theta_e}{\theta_o} \quad (12)$$

and then we design the circuit for infinite isolation at one (optional) frequency close to the center frequency of the device. The conditions for infinite isolation at the frequency f_r ($f_r = 1 \pm L = \lambda/4$) are:

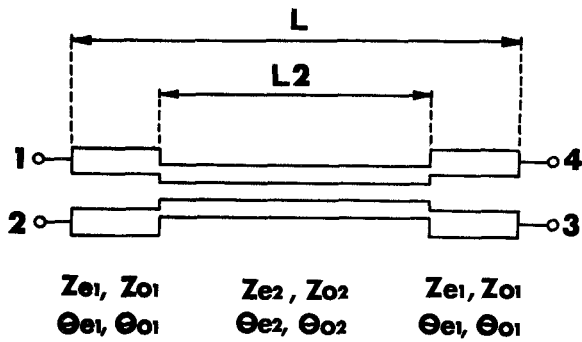


Figure 4 - The new coupler with compensating sections

$$\frac{1+(t_{1e}t_{2e})^2}{t_{1e}t_{2e}} - R_e - \frac{1}{R_e} = \frac{1+(t_{1o}t_{2o})^2}{t_{1o}t_{2o}} - R_o - \frac{1}{R_o} \quad (13)$$

$$\frac{t_{1e}^2+t_{2e}^2}{t_{1e}t_{2e}} + R_e + \frac{1}{R_e} = \frac{t_{1o}^2+t_{2o}^2}{t_{1o}t_{2o}} + R_o + \frac{1}{R_o}$$

$$Z_{e2} \cdot Z_{o2} = \frac{1}{R_e R_o} \frac{(1 - t_{1o}t_{2o}/R_o)(t_{1e} + R_e t_{2e})}{(t_{1o} + t_{2o}/R_o)(1 - R_e t_{1e}t_{2e})} \quad (14)$$

where: $R_e = Z_{e1}/Z_{e2}$ (15)
 $R_o = Z_{o1}/Z_{o2}$ (16)
 $t_{1e} = \tan(fr \cdot \theta_{e1})$ (17)
 $t_{2e} = \tan(fr \cdot \theta_{e2}/2)$ (18)
 $t_{1o} = \tan(fr \cdot \theta_{o1})$ (19)
 $t_{2o} = \tan(fr \cdot \theta_{o2}/2)$ (20)
 $\theta_e = 90 + \Delta\theta = 2\theta_{e1} + \theta_{e2}$ (21)
 $\theta_o = 90 - \Delta\theta = 2\theta_{o1} + \theta_{o2}$ (22)
 $\frac{L2}{L} = \frac{\theta_{e2}}{\theta_e} = \frac{\theta_{o2}}{\theta_o}$ (23)

By a straightforward calculation, we can eliminate R_e , R_o , Z_{e1} , Z_{o1} , Z_{e2} and Z_{o2} in equations 13 and 14 and use:

$$CPL1 = 20 \log [(Z_{e1}-Z_{o1})/(Z_{e1}+Z_{o1})] \quad (24)$$

$$CPL2 = 20 \log [(Z_{e2}-Z_{o2})/(Z_{e2}+Z_{o2})] \quad (25)$$

$$IMP1 = \sqrt{Z_{e1} \cdot Z_{o1}} \quad (26)$$

$$IMP2 = \sqrt{Z_{e2} \cdot Z_{o2}} \quad (27)$$

We observe that we have found independent parameters: $L2/L$, fr , $CPL1$ and $CPL2$ is a good choice. Equations 13 and 14 then give $IMP1$ and $IMP2$. As shown in Figure 5, there are two different realizations depending on whether the outer sections are tighter or looser coupled than the center section of the coupler.

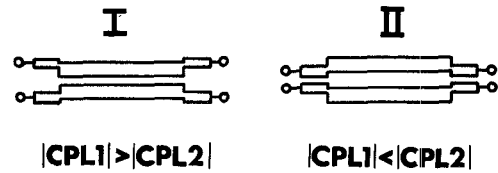


Figure 5 - Different Realizations of the Compensated Coupler

A useful relationship between the two cases $\theta_e > \theta_o$ and $\theta_e < \theta_o$ has been found. It has been shown that the same performance (except $|S_{11}|$ and $|S_{31}|$) is achieved if:

$$\frac{\theta_o}{\theta_e} (\theta_e > \theta_o) = \frac{\theta_e}{\theta_o} (\theta_e < \theta_o) \quad (28)$$

$$CPL1 (\theta_e > \theta_o) = CPL1 (\theta_e < \theta_o) \quad (29)$$

$$CPL2 (\theta_e > \theta_o) = CPL2 (\theta_e < \theta_o) \quad (30)$$

$$IMP1 (\theta_e > \theta_o) = 1/IMP1 (\theta_e < \theta_o) \quad (31)$$

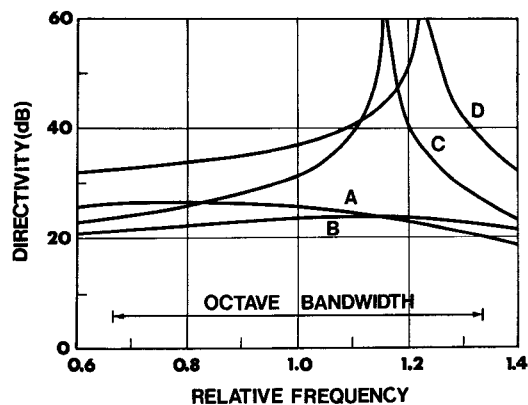
$$IMP2 (\theta_e > \theta_o) = 1/IMP2 (\theta_e < \theta_o) \quad (32)$$

To get an idea of the usefulness of the compensating method a number of calculations has been made. Some theoretical curves for 3 dB resp. 20 dB couplers are shown in Figure 6 resp. Figure 7 and the corresponding parameters are given in Table 2. The Table shows the case $\theta_e < \theta_o$ as well as the case $\theta_e > \theta_o$ since the same curves apply to both cases. The ratio $L2/L$ gives the length of the center section of the coupler compared to its total length. $L/(0.25\lambda_0)$ shows the total length of the coupler at the frequency of maximum coupling. $0.25\lambda_0$ corresponds to the frequency where the mean value of the total even and odd mode lengths equals 90 degrees. The minimum return loss and directivity has been calculated for a 2.3:1 bandwidth (octave bandwidth with margin). All the couplers in Table 2 except the single-section couplers with $IMP2 = 1$ have been optimized so the directivity is the same at the two edges of the band. For the compensated couplers this was done by varying the location of the directivity pole (parameter fr).

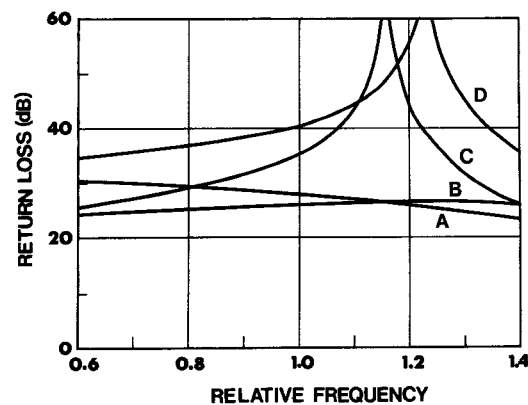
When we compare the coupling of a compensated coupler to that of a perfect coupler over an octave, we see that the difference is generally less than a few hundredths of a dB. The coupling maximum of a compensated coupler is moved in frequency for circuit I to a higher frequency (lengthening effect) and for circuit II to a lower frequency (shortening effect).

Curve	Type of Circuit	Center Freq. Coupling (dB)	θ_e/θ_o or θ_o/θ_e (%)	$\theta_e < \theta_o$		$\theta_e > \theta_o$		CPL1 (dB)	CPL2 (dB)	$\frac{L2}{L}$	$\frac{L}{0.25\lambda_0}$	Min. Ret. Loss (dB)	Min. Directivity (dB)
				IMP1 (ohm)	IMP2 (ohm)	IMP1 (ohm)	IMP2 (ohm)						
A	I	-2.73	90	---	1.000	---	1.000	---	-2.70	1.00	0.99	-23.1	-18.6
B		-2.70	90	---	0.939	---	1.065	---	-2.66	1.00	1.00	-24.5	-20.9
C		-2.70	90	0.593	0.955	1.686	1.047	-30.00	-2.66	0.90	1.10	-25.5	-22.7
D		-2.70	90	1.118	0.705	0.894	1.418	-1.88	-20.00	0.10	0.86	-34.6	-31.9
E	II	-19.38	98	---	1.000	---	1.000	---	-19.38	1.00	1.00	-52.5	-11.9
F		-19.38	98	---	0.885	---	1.130	---	-19.25	1.00	0.99	-18.4	-13.8
G		-19.38	98	0.390	0.929	2.564	1.076	-40.00	-19.21	0.90	1.09	-18.5	-16.7
H		-19.38	98	1.239	0.994	0.807	1.006	-6.02	-21.00	0.90	0.77	-40.5	-39.5

Table 2. Design data for compensated and uncompensated couplers with unequal even and odd mode lengths. Directivity and return loss are calculated for a 2.3:1 bandwidth.



(a)

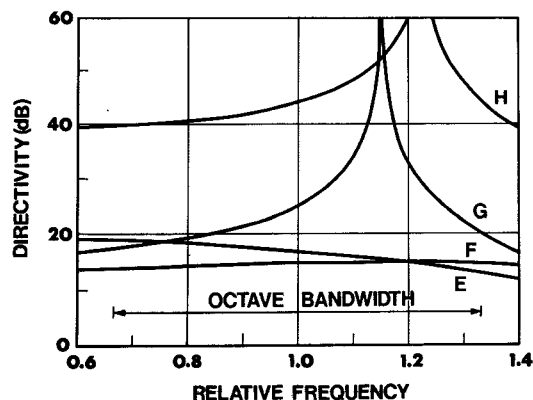


(b)

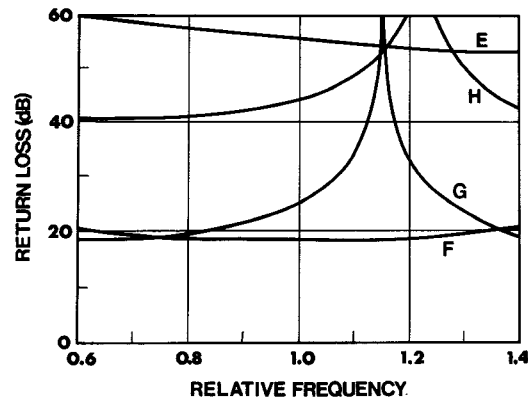
Figure 6 - Theoretical curves of couplers with a center frequency coupling of 2.70 dB. The parameters are found in Table 2.

a. Directivity

b. Return Loss



(a)



(b)

Figure 7 - Theoretical curves of couplers with a center frequency coupling of 19.38 dB. The parameters are found in Table 2.

a. Directivity

b. Return Loss

We find that the widest bandwidth for circuit I is obtained when the outer sections are as short as possible and have low coupling. The limiting factor is that IMP_1 goes very low ($\theta_e < \theta_0$) or high ($\theta_e > \theta_0$). For stripline couplers ($\theta_e < \theta_0$), the outer sections take the form of tabs and has been used for years at Anaren Microwave, Inc., (and some other industries), but no quantitative theory has been presented. A comparison between the new theory and experimental circuits shows good agreement.

The widest bandwidth for circuit II is achieved when the outer sections are short and the center section has low coupling. The limiting factor is the coupling in the outer sections. The directivity improvement over a 2.3:1 bandwidth is generally better than for circuit I. Circuit II is particularly effective when the overall coupling is 10 dB or less.

Conclusion

A new method to measure the parameters of CTL-couplers has been presented including explicit formulas for the necessary calculations. The method has been very useful in practice for the correction of couplers with $Z_e \neq 1/Z_0$ and/or $\theta_e \neq \theta_0$. A new circuit has been developed for improvement of the directivity of CTL-couplers with different even and odd mode lengths. Explicit formulas were given for the design of couplers with a directivity pole at an arbitrary frequency. The presented curves show that a large improvement can be achieved over a wide bandwidth.

Acknowledgement

The author wishes to thank Carl W. Gerst, Jr., Executive Vice-President at Anaren Microwave, Inc., for many discussions during this work and for the permission to publish the results.

References

1. R. Lagerlöf, S. Rehnmark, "VHF - antenna feeder power divider," Microwave Journal, Euro-Global Edition, July, 1975.
2. S. Rehnmark, "Theory and application of microwave couplers, phase shifters, and power dividers," Doctor thesis at School of Electrical Engineering, Chalmers University of Technology, Gothenburg, Sweden, Technical Report No. 62, April, 1976, Paper D, pp. 6-8.
3. S. Rehnmark, "Meander-folded coupled lines," to be published in IEEE Trans. Microwave Theory Tech.
4. A. Podell, "A high directivity coupler technique," G-MTT 1970, International Microwave Symposium, May, 1970, pp. 33-36.
5. C. Buntschuh, "Octave-bandwidth, high directivity microstrip coupler," RADC-TR-73-396, Contract F30602-72-C-0282, AD 777320, January, 1974.